

New Math Bits Unit for 7th Grade: Integers

In the current Math Bits curriculum, integers are introduced from an algebraic perspective. First, students learn algebraic language and how to work with expressions involving letters in the *Algebraic Language* unit. Then, integers are introduced through algebra in the *Integers and Equations* unit.

This approach follows the work of [educational experts](#) Eva Cid and Pablo Beltrán-Pellicer from the University of Zaragoza, whose research in math education has inspired parts of our program. The goal is to avoid certain common difficulties that can arise with other teaching approaches.

After implementing this sequence in classrooms, we received valuable feedback from teachers. As a result, we have developed a new *Integers* unit that offers a different instructional approach—also based on research in math education—and equally effective (see its rationale below).

You can now access this new unit in your intranet. For now, only the “Explain” lessons are available; the rest will be added between June and September 2025.

The previous *Algebraic Language* and *Integers and Equations* units will remain available for those who wish to continue using them.

Instructional Approach

In the new *Integers* unit, as in all Math Bits materials, the focus is on building a conceptual understanding of new ideas and procedures rather than just practicing calculations.

Different models are used to introduce integers. Each model is more or less helpful depending on which property is being explored, but taken together, they offer students a deeper, more complete understanding of how integers work.

Broadly speaking, we use three types of models:

1. Arithmetic model
2. Geometric model
3. Neutralization model

Arithmetic Model

Integers are presented as an extension of the natural numbers to make sense of subtractions where the subtrahend is greater than the minuend (e.g., $3 - 5 = -2$).

We use inductive reasoning to generalize properties of natural numbers and, from there, to derive the properties of integers and the operations involving them.

For example: If we subtract increasingly larger numbers from a given number, the result becomes smaller. So, we can say that -4 is less than -3 because $-4 = 0 - 4$ and $-3 = 0 - 3$.

		The subtracted number increases		The result is smaller	
$4 - 0 = 4$	$3 - 0 = 3$	$2 - 0 = 2$	$1 - 0 = 1$	$0 - 0 = 0$	
$4 - 1 = 3$	$3 - 1 = 2$	$2 - 1 = 1$	$1 - 1 = 0$	$0 - 1 = -1$	
$4 - 2 = 2$	$3 - 2 = 1$	$2 - 2 = 0$	$1 - 2 = -1$	$0 - 2 = -2$	
$4 - 3 = 1$	$3 - 3 = 0$	$2 - 3 = -1$	$1 - 3 = -2$	$0 - 3 = -3$	
$4 - 4 = 0$	$3 - 4 = -1$	$2 - 4 = -2$	$1 - 4 = -3$	$0 - 4 = -4$	
$4 - 5 = -1$	$3 - 5 = -2$	$2 - 5 = -3$	$1 - 5 = -4$	$0 - 5 = -5$	

Another example: If multiplication by a positive number is understood as repeated addition, then multiplication by a negative number can be seen as repeated subtraction. This helps introduce the rule of signs for multiplication.

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$(-3) \cdot (-5) = +15$

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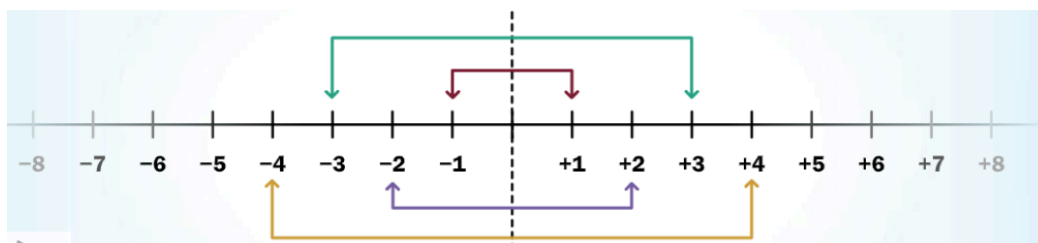
$-(-5) - (-5) - (-5) = +15$

Multiplying -5 by -3 is the same as subtracting -5 3 times.

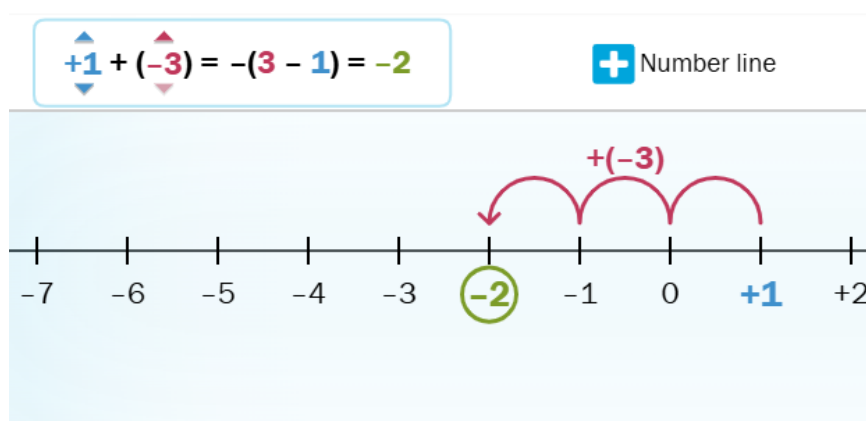
Geometric Model

Integers are shown on a number line. This line is oriented and uses an arbitrary point to represent 0. Positive numbers appear to the right of 0, and negative numbers to the left.

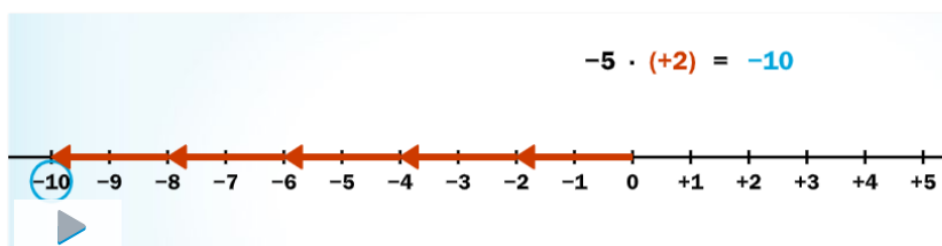
The number line offers a visual way to explain how integers are ordered (the farther to the right, the greater the number, and vice versa) and how absolute value represents the distance from an integer to 0.



Addition and subtraction of integers can be shown as movements along the number line.



Multiplying by a positive number can be seen as stretching a directed segment. Multiplying by a negative number is seen as stretching (dilating) and flipping (inverting) the segment.

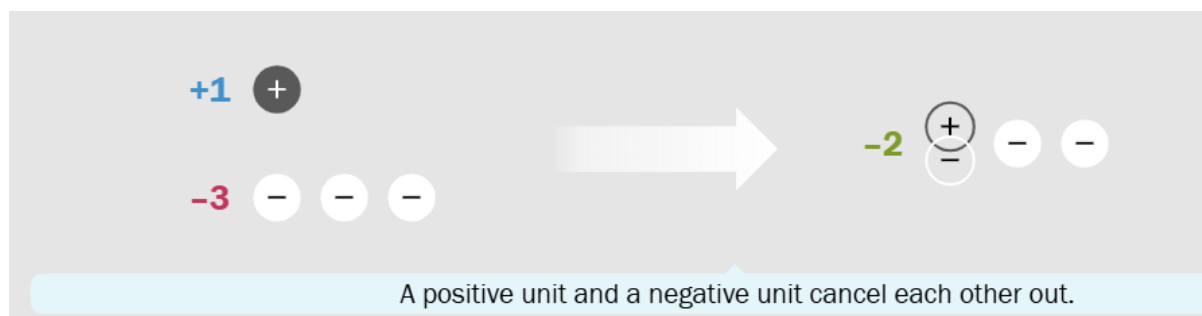


Neutralization Model Using Integer Tiles

This model uses two-color tiles to represent positive (+1) and negative (−1) units. A pair of opposite tiles cancels out, or neutralizes. Therefore, adding or removing pairs of opposite tiles does not change the total.

This means there are many different ways to represent the same integer. For example, +4 can be shown as 4 positive tiles, or as 5 positives and 1 negative, 6 positives and 2 negatives, and so on.

With these multiple representations, integer addition and subtraction can be understood as adding or removing tiles.



References

This new unit is based on the following educational research:

- González, J. L., [et al.]. (1990). *Números enteros* (Colección Matemáticas: cultura y aprendizaje). Editorial Síntesis.
- Lytle, P. A. (1992). *Use of a neutralization model to develop understanding of integers and of the operations of integer addition and subtraction* (Master's thesis, Concordia University).
- Hayes, R. L. (2019). *Teaching Negative Number Operations: A Comparative Study of the Neutralization Model Using Integer Tiles*. ResearchGate.

Unit Structure

1. Engage: Numbers With Two Meanings

Students are introduced to everyday, real-world situations where signed numbers are used.

2. Explore: Positive and Negative Points

Students investigate a game involving point gains and losses to see what happens when subtracting from zero. They explore a second game involving cards where positive and negative points can be won or lost to better understand integer addition and subtraction.

3. Explain: Integers

- Students are introduced to integers.
- They learn about absolute value and opposites.
- They compare and order integers.

- They begin using the number line to represent them.

4. Explain: **Adding and Subtracting Integers**

- Students learn to add and subtract integers using different models.
- They explore different meanings of subtraction: as taking away and as finding a difference.
- They use differences between two integers to classify numbers.
- They use differences to calculate changes in quantities.

5. Explore: **Dilations and Inversions**

Students play a mini-game in which objects are stretched or flipped to reach a goal. This introduces multiplication by positive integers as stretching, and multiplication by negative integers as stretching and flipping. stretching and flipping.

6. Explain: **More Operations With Integers**

- Students learn how to multiply integers using different models.
- They understand integer division as the inverse of multiplication.
- They are introduced to powers with integer bases and natural exponents.
- They explore how properties of integers differ from those of natural numbers.

7. Evaluate: **Evaluation - Integers**

ADDITIONAL LESSONS

8. Elaborate: **Playing With Integers**

Students are asked to design a game where integers play a meaningful role.

You can access the new Math Bits 2025 proposal [here](#).

If you have any questions, please reach out to your school's educational advisor.